Thermal Postbuckling Behavior of Laminated Composite Plates

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Introduction

THE subject of buckling and postbuckling in laminated composite plates is a rapidly developing one; as a consequence, large number of publications on the subject have appeared in the recent years. A comprehensive summary of the state-of-art in buckling and postbuckling has been compiled by Chia,[1] Kapania,[2] and Leissa.[3] The postbuckling response of composite plates subjected to mechanical edge compression and/or shear was investigated by Harris,[4] Turvey and Wittrick,[5] Chia and Prabhakara,[6] Stein,[7] Jensen and Legace,[8] and Dawe and Lam.[9] Most of these studies are based on series solutions. The subject of thermal postbuckling of laminated plates is presently under the attention of many researchers as is evident from the recent publications by Dumir,[10] Chen and Chen,[11] Chang,[12] and Tauchert and Huang.[13]

It is well known that the presence of bending-extension coupling in unsymmetrically laminated plates gives rise to bending curvatures under the action of pure in-plane loading, no matter how small these loads may be. Hence, the existence of bifurcation-type instability is questionable. Most of the researchers have solved the homogeneous partial differential equations (governing equations) as an eigenvalue problem even for unsymmetrically laminated plates without showing any apparent concern towards the validity of their results, while a few others have considered this aspect. Among those who considered this aspect are Harris,[4] Jensen and Legace,[8] and Dawe and Lam.[9] Harris[4] demonstrated the existence of bifurcation buckling in regular antisymmetric angle-ply plates subjected to uniaxial and biaxial compression and stated that it does not exist for generally unsymmetric angle-ply laminates. Chia[1] and Dawe and Lam[9] stated that due to the presence of material coupling, eigenvalue-type buckling will not usually occur since out-of-plane displacements develop from the commencement of in-plane loading. In such circumstances, there is no postbuckling behavior as such but it is reasonable to refer to this behavior at relatively large loads as pseudopostbuckling behavior.

The literature review reveals that the postbuckling response of unsymmetrically laminated plates is considerably more complex and requires careful attention. Most of the investigators who have contributed in this area have not taken into account the edge moments developed due to the non-coincidence of the midplane and neutral planes. The existence of secondary instabilities from the postbuckling path also has received little attention of the researchers. Thus, there is a need to investigate the applicability of the response curves obtained by them.

Formulation

The displacement field of rectangular shear deformable plates can be expressed as

$$\bar{u}(x, y, z) = u(x, y) - z w_b'(x, y) - \frac{4z^3}{t^2} w_s'(x, y) \tag{1a}$$

$$\bar{v}(x, y, z) = v(x, y) - z w_b^o(x, y) - \frac{4z^3}{t^2} w_s^o(x, y) \tag{1b}$$

$$w(x, y, z) = w_b(x, y) + w_s(x, y) \tag{1c}$$

The nonlinear strain-displacement relations, which include thermal strains and allow parabolic variation of transverse shear strain with vanishing of transverse shear stresses at the top and bottom surface of the plate, may be expressed as

$$\epsilon_x = u' + \frac{1}{2}(w_b' + w_s')^2 - z w_b'' - \frac{4z^3}{3t^2} w_s'' - \alpha_x \Delta T \tag{2a}$$

$$\epsilon_y = v^o + \frac{1}{2}(w_b^o + w_s^o)^2 - z w_b^{oo} - \frac{4z^3}{3t^2} w_s^{oo} - \alpha_y \Delta T \tag{2b}$$

$$\gamma_{xy} = u^o + v' + (w_b' + w_s')(w_b^o + w_s^o) - 2z w_b'^o - \frac{8z^3}{3t^2} w_s'^o - \alpha_{xy} \Delta T \tag{2c}$$

$$\gamma_{xz} = \left(1 - \frac{4z^2}{t^2}\right) w_s', \qquad \gamma_{yz} = \left(1 - \frac{4z^2}{t^2}\right) w_s^o \tag{2d}$$

where ϵ_x, ϵ_y, and γ_{xy} are the in-plane normal and shear strains, and γ_{xz} and γ_{yz} are the transverse shear strains. u and v are the displacements in the x and y directions, respectively. The transverse displacement in the z direction is represented as sum of the components w_b and w_s. α_x, α_y, α_{xy}, and ΔT are the coefficients of thermal expansion and constant temperature difference. $(\ )'$ and $(\ )^o$ represent partial differentiation with respect to x and y.

The strain energy of the plate can be expressed in terms of the field variables u, v, w_b, w_s, and their derivatives. The variation of the strain energy will lead to four governing equations and six natural/essential boundary conditions at each edge. While the essential boundary conditions imply the specification of u, v, w_b, w_b' / or w_b^o, w_s' / or w_s^o, the corresponding natural boundary conditions mean specification of in-plane normal stress resultant, in-plane shear stress resultant, transverse shear force, higher-order transverse shear force, stress couples, and higher-order stress couples. These governing equations are solved by employing a finite element approach. The four-node rectangular C^1 continuous element proposed by Bogner et al.[14] is extended for this purpose. The domain of the rectangular plate is assumed to be divided into a number of four-node rectangular elements. The element has 14 degrees of freedom per node, namely u, u', u^o, v, v', v^o, w_b, w_b', w_b^o, $w_b'^o$, w_s, w_s', w_s^o, $w_s'^o$. The shape functions employed to describe these nodal variables are given in Ref. 14.

Using Eq. (2) and following the standard procedure,[15] the nonlinear finite element equations are derived as

$$\left\{[K_o] + \frac{1}{2}[N_1] + [N_2]\right\}\{\delta\} + \lambda[K_{gm}]\{\delta\} = \{F\} \tag{3}$$

where $[K_o]$, $[N_1]$, $[N_2]$, and $\{F\}$ are the linear stiffness matrix independent of field variables, stiffness matrix linearly depending on field variables, stiffness matrix quadratically depending on field variables, and load vector, respectively.

The stress state developed in the plate very much depends on the lay-up and boundary conditions, and it is this stress state which is responsible for plate buckling rather than the applied one. In the present work, postbuckling analysis is carried out through the following steps.

Step 1. Linear prebuckling analysis is carried out by neglecting nonlinear terms.
Step 2. Geometric stiffness matrix $[K_{gm}]$ is constructed.
Step 3. Critical temperature is evaluated, solving Eq. (3) as an eigenvalue problem. (Matrices $[N_1]$ and $[N_2]$ are neglected in first iteration and $\{F\} = \{0\}$.)

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Step 4. The eigenvector is scaled up by multiplying with a constant factor. This constant factor represents maximum deflection of the plate.
Step 5. Nonlinear matrices $[N_1]$ and $[N_2]$ are constructed based on scaled up eigenvector. $[K_{gm}]$ is retained as in the first iteration.
Step 6. Linear eigenvalue problem with nonlinear matrices is solved to compute postbuckling loads/temperatures and mode shapes.
Step 7. Steps 4–7 are repeated until postbuckling load obtained from the subsequent two iterations is within specified tolerance.

Unsymmetrically laminated composite plates which do not exhibit bifurcation-type instability, their postbuckling response is a hypothetical concept. It is generally believed that these pseudopostbuckling curves will be close to true load-deflection curves at large loads.[9] To examine this aspect, the nonlinear prebuckling Eq. (3) is solved iteratively by following the same scheme as explained earlier (by neglecting K_{gm}). The only difference is that in the place of an eigenvalue problem, nonlinear simultaneous equations are solved by matrix inversion.

Results and Discussions

Based on the nonlinear finite element Eq. (3), numerical results for the critical load or temperature and postcritical to critical temperature ratios are presented for rectangular composite plates. The element developed herein is tested for many standard problems and is found to yield very accurate results. The results presented in this section are obtained using a 4×4 mesh over the whole plate. This idealization was chosen on the basis of a convergence study. The details of this convergence and comparison study are omitted for the sake of brevity.

The mechanical properties used in this section are as follows:

Material I:

$$E_L/E_T = 40, \qquad G_{LT}/E_T = G_{LZ}/E_T = 0.5$$

$$G_{TZ}/E_T = 0.2, \qquad \nu_{LT} = 0.25, \qquad \alpha_T/\alpha_L = 10$$

Material II:

$$E_L/E_T = 25, \qquad G_{LT}/E_T = G_{LZ}/E_T = 0.5$$

$$G_{TZ}/E_T = 0.2, \qquad \nu_{LT} = 0.25, \qquad \alpha_T/\alpha_L = 10$$

The following edge constraints are prescribed in the numerical examples presented in this section:

Hinged-hinged (HHHH):

$$u = v = w_b = w_s = 0, \qquad \text{at } x = 0, a \text{ and } y = 0, b$$

Fixed-fixed (FFFF):

$$u = v = w_b = w_s = w_b' = w_s' = 0, \qquad \text{at } x = 0, a$$

$$u = v = w_b = w_s = w_b^o = w_s^o = 0, \qquad \text{at } y = 0, b$$

Figure 1 gives the variation of postbuckling temperature to critical temperature ratio with plate center deflection to thickness ratio for four-layered symmetric and antisymmetric angle-ply plates. It may be noted that antisymmetric angle-ply plates have higher buckling resistance compared to their symmetric counterparts. It may be noted that such plates do exhibit bifurcation instability; the bifurcation point or temperature in these plates is represented by the intersection of postbuckling curves with the ordinate. The ordinate in such cases represent the trivial prebuckling path. It may be observed that the plate center deflection increases with the increase in the temperature and then suddenly there appears a discontinuity in the curves. These discontinuities correspond to the secondary instability from the postbuckling path. In the case of symmetrically laminated plates, as the load-deflection curve reaches the point B, two possibilities exist; 1) if the temperature is maintained constant then the plate may exhibit snap-through-type phenomenon and likely to meet the extended curve CD. The jump is associated with a change in mode shape. 2) if the deflection is controlled then the load abruptly drops to point C and moves along the path CD. A similar phenomenon is exhibited by the antisymmetrically laminated plates.

A similar conclusion can be drawn from the study presented in Fig. 2, wherein postbuckling response curves for four-layer symmetric and antisymmetric angle-ply plates with $\Theta = 45$ deg are plotted. It may be observed that symmetric and antisymmetric lay-ups considered in this study result in higher buckling loads compared to the one's considered in Fig. 1.

The effect of number of layers and lay-up sequence on the postbuckling response curves for a square cross-ply plate is presented in Fig. 3. It is interesting to note that even though bending curvature does appear in the case of square antisymmetric cross-ply plates, they still exhibit bifurcation instability. This is once again revealed by the intersection of postbuckling curves and the ordinate representing prebuckling behavior. As expected, the critical temperature increases with the increase in number of layers and further symmetric lay-up yields the maximum buckling load. It may be observed that four-layer antisymmetric cross-ply plates are capable of taking much higher loads beyond the bifurcation point and secondary instability sets in at relatively very high load. However, in

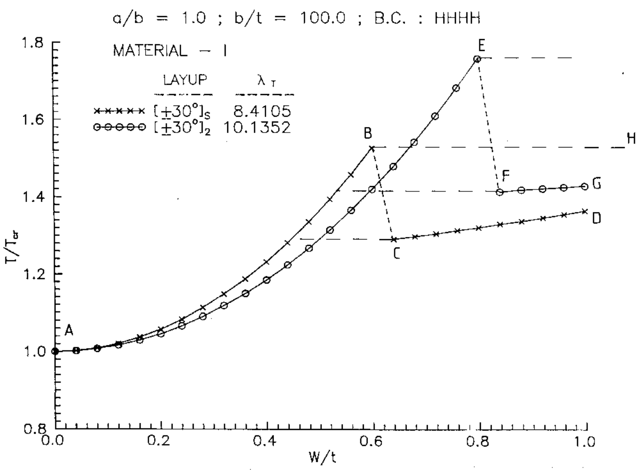

Fig. 1 Effect of lay-up sequence on the postcritical temperature of simply supported angle-ply plates.

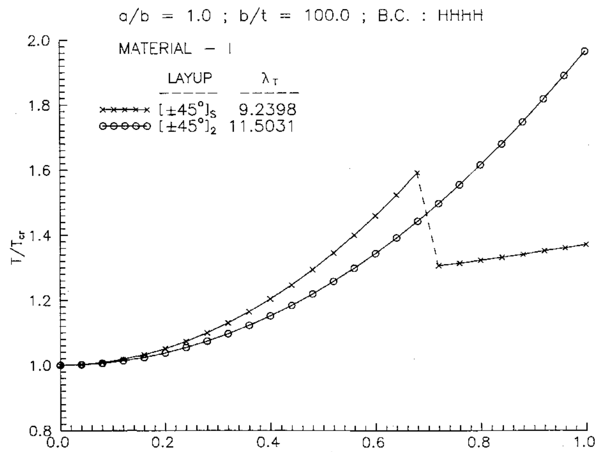

Fig. 2 Effect of lay-up sequence on the postcritical temperature of simply supported angle-ply plates.

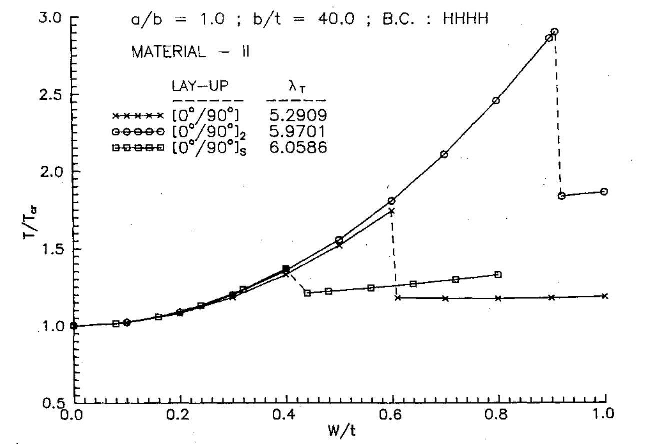

Fig. 3 Effect of lay-up sequence on the postcritical temperature of simply supported cross-ply plates.

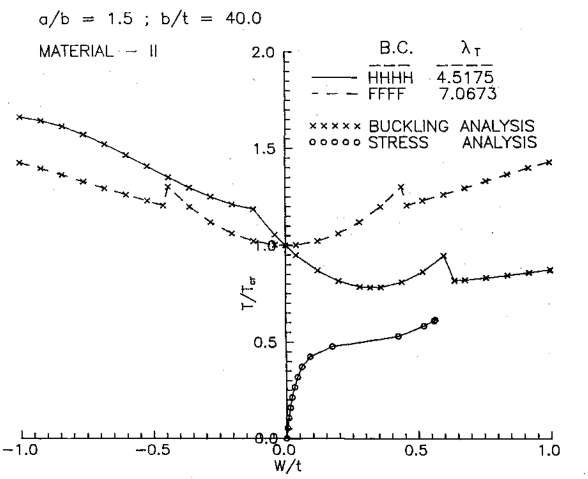

Fig. 4 Postcritical temperature and temperature-deflection curve for rectangular cross-ply plate.

the case of two-layer antisymmetric and four-layer symmetric cross-ply plates, these secondary instabilities occur at relatively very small loads.

The variation of plate center deflection with temperature for rectangular antisymmetric cross-ply plates with hinged-hinged (HHHH) and fixed-fixed (FFFF) boundary conditions is plotted in Fig. 4. It may be noted that bifurcation instability does take place for such plates when all of the edges are fixed and it is represented by intersection of postbuckling curve with the prebuckling equilibrium path (temperature axis). However, in case of such plates with hinged-hinged edges, bifurcation cannot take place, because the prebuckling equilibrium path is not represented by temperature axis but by a curve included in this figure. It may be observed that this curve does not intersect the pseudopostbuckling curve traced herein, thus bifurcation cannot occur in such plates. Furthermore, it may not be correct to refer to this pseudopostbuckling curve even at large loads as the true nonlinear prebuckling load-deflection curve is far away from this pseudopostbuckling curve. The prebuckling load-deflection curve is not traced beyond a certain value of temperature load due to convergence problems. The convergence problems in the nonlinear prebuckling analysis may be attributed to the expected deformation distribution changes beyond this load (refer corresponding pseudopostbuckling curve), which could not be captured with the iterative solution scheme employed herein. As bifurcation instability does not take place in such plate, the buckling of such plates is thus characterized by rapid growth of deformation as the critical load/temperature is approached. The nonlinear prebuckling load-deflection curves reveal that such plates will buckle at nearly half of the load, which is computed through an eigenvalue analysis. Therefore, the critical load/temperature for unsymmetrically laminated plates which do not exhibit bifurcation should be evaluated through nonlinear stress analysis only.

Conclusions

Thermal buckling and postbuckling behavior of shear deformable laminated composite plates is investigated by employing a four-node rectangular C^1 continuous finite element. The investigation reveals that the postbuckling path may not remain stable throughout. It is shown that secondary instabilities coupled with changes in the spatial deformation do take place from the postbuckling path. Simply-supported square antisymmetric cross-ply plates wherein bending curvatures appear from the commencement of loading are shown to exhibit bifurcation instability. It is also found that though the bifurcation loads for two-layer antisymmetric cross-ply plates are smaller compared to four-layer symmetrically laminated plates, in the case of four-layer plates secondary instabilities set in at a very early stage. The rectangular antisymmetric cross-ply plates with fixed edges are also found to exhibit bifurcation. However, for these plates with hinged-hinged edge conditions, bifurcation does not take place. The only way to correctly predict the buckling of such plates is to carry out a nonlinear stress analysis. It is shown that the critical loads predicted through an eigenvalue analysis in such cases could be twice compared to the actual buckling load.

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